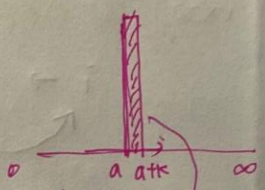
$f_k(t-a) \begin{cases} \frac{1}{k} & (a \le t \le a+k) \\ 0 & \text{이외} \end{cases}$

→ δ = delta ↓

☆ $\delta(t-a) \begin{cases} \infty & (t=a) \\ 0 & \end{cases}$ ☆

$= \lim_{k\to 0} f_k(t-a)$



$\therefore \boxed{\int_0^\infty f_k(t-a)\,dt = \int_a^{a+k} \frac{1}{k}\,dt = 1}$

② $\boxed{f_k(t-a) = \frac{1}{k}[u(t-a) - u(t-(a+k))]}$

$\downarrow \mathcal{L}$

$\mathcal{L}(f_k(t-a)) = \frac{1}{ks}[e^{-as} - e^{-(a+k)s}]$

$= e^{-as}\frac{(1-e^{-ks})}{ks}$

$\lim_{k\to 0} \frac{e^{-as}(1-e^{-ks})}{ks} \to \lim_{k\to 0} \frac{se^{-ks}}{s} = 1$

$\therefore \boxed{\mathcal{L}\{\delta(t-a)\} = e^{-as}}$

③ Sampling 특성

$\int_0^\infty g(t)\,\delta(t-a)\,dt = g(a)$ 임펄스가 존재하는 시점 (t=a)의 함숫값..

임의의 함수 × 임펄스 함수

☆ ↗ $e^{-as} = e^{-s}$

ex2) $y'' + 3y' + 2y = \delta(t-1)$ 시간 t=1에서의 단위 충격,,

↓

$s^2Y + 3sY + 2Y = e^{-s}$

$\therefore Y(s) = \frac{e^{-s}}{(s+1)(s+2)} = \left(\frac{1}{s+1} - \frac{1}{s+2}\right)e^{-s}$

↓

$y(t) = \mathcal{L}^{-1}(Y) = \begin{cases} 0 & (0<t<1) \\ \ldots \end{cases}$

6.5 합성곱

① Convolution.

$(f*g)(t) = \int_0^t f(\tau)\,g(t-\tau)\,d\tau$

② 특성
1) $f*g = g*f$
2) $f*(g_1+g_2) = f*g_1 + f*g_2$
3) $(f*g)*v = f*(g*v)$
4) $f*0 = 0*f = 0$
5) $f*1 \ne 1$

☆☆ Thm) $\mathcal{L}(f*g) = \mathcal{L}(f)\mathcal{L}(g)$

ex1) $H(s) = \frac{1}{(s-a)s}$

$\mathcal{L}^{-1}\left(\frac{1}{s-a}\right) = e^{at}$, $\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1$

$h(t) = e^{at} * 1 = \int_0^t e^{a\tau}\cdot 1 \cdot d\tau = \frac{1}{a}(e^{a\tau} - 1)$

ex2) $H(s) = \frac{1}{(s^2+w^2)^2}$

$h(t) = \frac{\sin wt}{w} * \frac{\sin wt}{w}$

$= \int_0^t \left(\frac{\sin w\tau}{w}\right)\cdot\left(\frac{\sin w(t-\tau)}{w}\right) d\tau$ ☆

$= \frac{1}{w^2}\int_0^t (\sin w\tau)(\sin w(t-\tau))\,d\tau$

$= \frac{1}{2w^2}\left[-\tau\cos wt + \frac{\sin w\tau}{w}\right]_{\tau=0}^{t}$

$= \frac{1}{2w^2}\left[-t\cos wt + \frac{\sin wt}{w}\right]$

② Second-shift Theorem

$\tilde{f}(t) = f(t-a)u(t-a) = \begin{cases} 0 & (t<a) \\ f(t-a) & (t>a) \end{cases}$

$\downarrow \mathcal{L}$ $\quad \mathcal{L}^{-1}_{s \to t}$

$e^{-as}F(s)$

③ Shifting Terms.

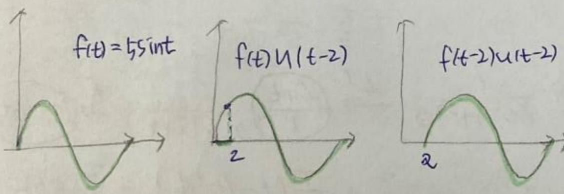
- S-domain shift

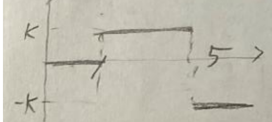
$T: e^{at} f(t) \longrightarrow S: F(s-a)$

- Time-domain shift

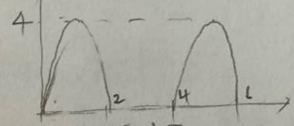
$S: e^{-as}F(s) \longrightarrow T: f(t-a)u(t-a)$

T-domain으로 변환 시,
u(t) 함수 필요하다.





$K[u(t-1) - 2u(t-4)]$



$4\sin(\frac{\pi}{2}t)[u(t) - u(t-2)]$

EX1) $f(t) = \begin{cases} 2 & (0<t<1) \\ \frac{t^2}{2} & (1<t<\frac{\pi}{2}) \\ \cos t & (t>\frac{\pi}{2}) \end{cases}$

Step 1

$\therefore f(t) = 2(1 - u(t-1)) + \frac{t^2}{2}(u(t-1) - u(t-\frac{\pi}{2}))$ ($1 = u(t)$)

$+ (\cos t)(u(t-\frac{\pi}{2}))$

⇒ u(t)를 이용해, 주어진 구간에서만 함수가 존재하도록 표현.

Step 2 ① $\mathcal{L}\{2(1 - u(t-1))\}$ ($1 = u(t)$)

$= \frac{2}{s} - \frac{2}{s}e^{-s}$

$= (\frac{1}{s^3} + \frac{1}{s^2} + \frac{1}{2s})e^{-s}$

③ $\mathcal{L}\{\frac{t^2}{2}u(t-\frac{\pi}{2})\} = \mathcal{L}\{(\frac{1}{2}(t-\frac{\pi}{2})^2 + \frac{\pi}{2}(t-\frac{\pi}{2}) + \frac{\pi^2}{8}) u(t-\frac{\pi}{2})\}$

$= (\frac{1}{s^3} + \frac{\pi}{2s^2} + \frac{\pi^2}{8s})e^{-\frac{\pi}{2}s}$

④ $\mathcal{L}\{(\cos t)u(t-\frac{\pi}{2})\} = \mathcal{L}\{-\sin(t-\frac{\pi}{2})u(t-\frac{\pi}{2})\}$

$= -\frac{1}{s^2+1}e^{-\frac{\pi}{2}s}$

$\therefore \mathcal{L}(f) = ① + ② + ③ + ④$

$* \boxed{\mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}}$

ex2)

$F(s) = \frac{e^{-s}}{s^2+\pi^2} \quad \frac{e^{-2s}}{s^2+\pi^2} \quad \frac{e^{-3s}}{(s+2)^2}$ (−1, −2, −3)

$\mathcal{L}^{-1} \downarrow$

$\frac{1}{\pi}\sin\pi(t-1)\,u(t-1)$

$\frac{1}{\pi}\sin\pi(t-2)u(t-2)$

$(t-3)e^{-2(t-3)}u(t-3)$

ex3) $V_0[u(t-a) - u(t-b)]$ 입력

$RI(s) + \frac{I(s)}{sC} = \frac{V_0}{s}[e^{-as} - e^{-bs}]$

$\boxed{I(s) = \frac{V_0}{(R+\frac{1}{sC})s}[e^{-as} - e^{-bs}]}$

$\downarrow$

$F(s) = \frac{\frac{V_0}{R}}{s + \frac{1}{RC}}$

$\downarrow$

$\mathcal{L}^{-1}(F(s)) = \frac{V_0}{R}e^{-\frac{t}{RC}}$

$i(t) = \mathcal{L}^{-1}(I(s)) = \mathcal{L}^{-1}\{e^{-as}F(s)\} - \mathcal{L}^{-1}\{e^{-bs}F(s)\}$

$\boxed{= \frac{V_0}{R}[e^{-\frac{t-a}{RC}}u(t-a) - e^{-\frac{t-b}{RC}}u(t-b)]}$

$F_B = \frac{\mu_0 I_1 I_2}{2\pi r} l$

3. 앙페르의 법칙

$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

$\boxed{\Phi_B = \int \vec{B} \cdot d\vec{A}}$

$[Wb] = [T \cdot m^2]$

① 직선도선

$B = \frac{\mu_0 I}{2\pi r} \quad (R \le r)$

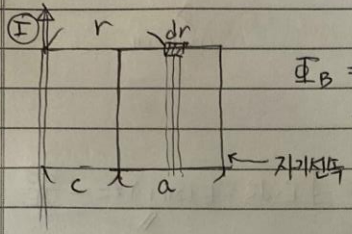
$B = \frac{\mu_0 I r}{2\pi R^2} \quad (R > r)$

② 토로이드

$B = \frac{\mu_0 N I}{2\pi r}$

③ 솔레노이드

$B = \mu_0 n I = \mu_0 \frac{N}{l} I$



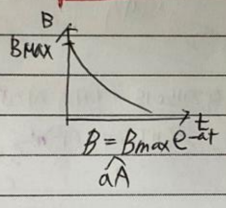
$\Phi_B = \int \vec{B} \cdot d\vec{A}$

$= \left(\frac{\mu_0 I}{2\pi}\right) \int_c^{c+a} \frac{1}{r} (b\,dr)$ 직선.

〈Ch30 패러데이〉

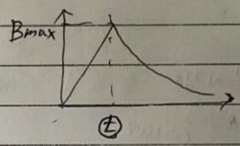
1. 패러데이 유도

$\boxed{\varepsilon = - \frac{d\Phi_B}{dt} = - \frac{d}{dt}(NBA\cos\theta)}$



$B = B_{max} e^{-at}$

dA



$B = a(AB_{max}) e^{-a(t-t)}$

2. 운동 기전력

① 자기장 내에서 도체의 움직임에 의해 유도되는 기전력 → 전하 q, 전류 I ..

$\Phi_B = B \cdot A = Blx$

$\varepsilon = -\frac{d\Phi_B}{dt} = -Blv$

$I = \frac{|\varepsilon|}{R} = \frac{Blv}{R}$

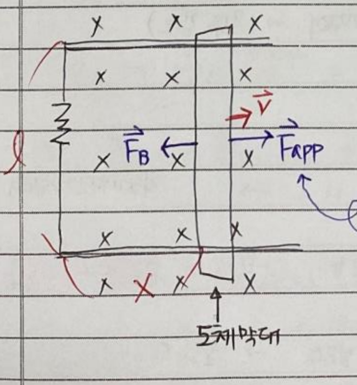
막대 속도

$V = V_i e^{-\frac{t}{\tau}}$

$X \sim V_i$

$\sim R$

$\sim \frac{1}{B^2}$



- 도체막대 내 전하 q에 미치는 자기력

$\vec{F}_B = q\vec{v} \times \vec{B} = qvB$

- 전하 q가 ⊕B를 돌 때 자기력이 하는 일

$W = \vec{F}_B \cdot \vec{l} = qvBl$

★ 유도기전력

$\varepsilon = \frac{W}{q} = \frac{-W}{q} = -vBl$

- 유도전류

$I = \frac{\varepsilon}{R} = \frac{Blv}{R}$

- 도체 막대에 흐르는 전류 I에 미치는 자기력

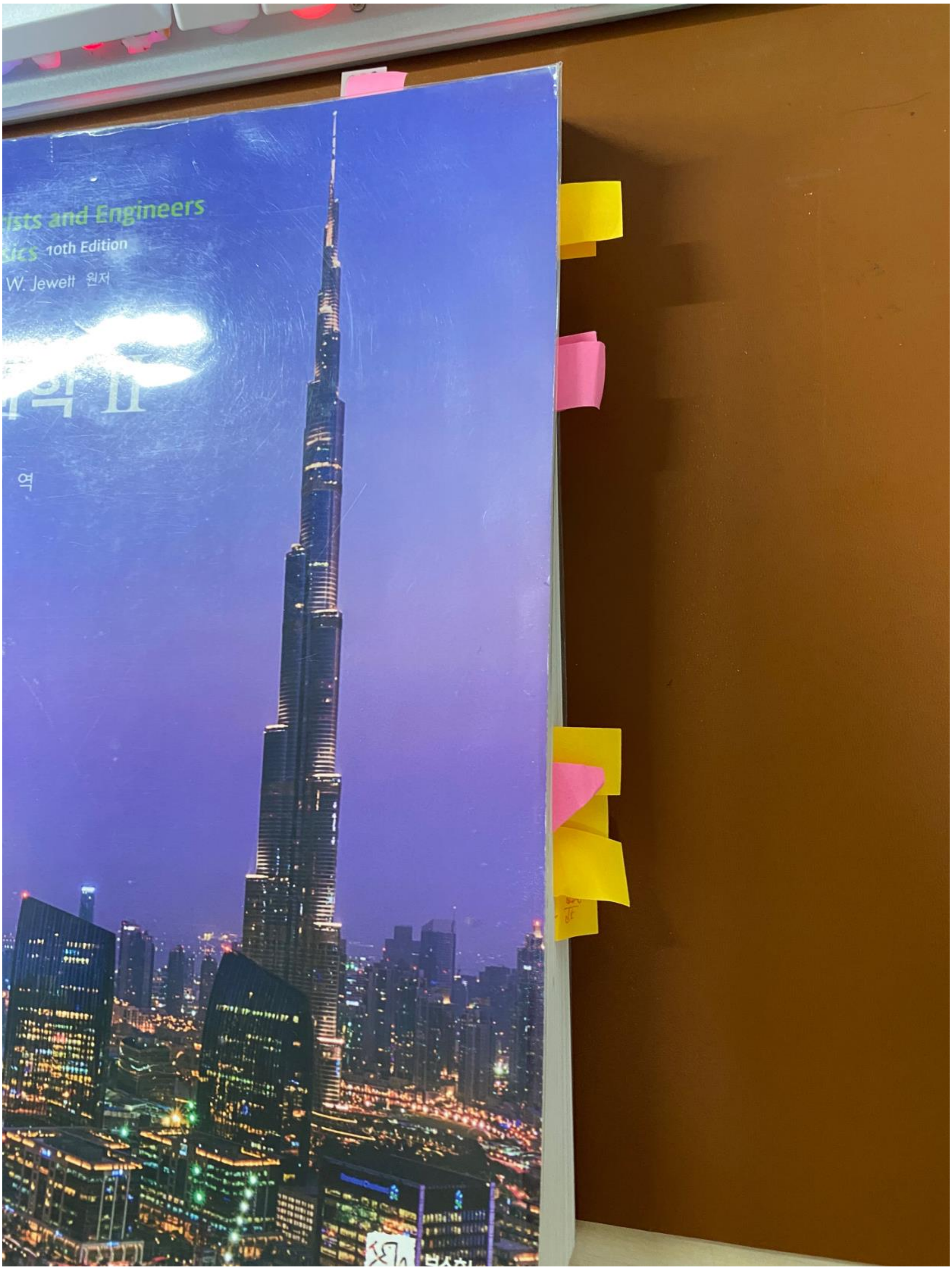
$\vec{F}_B = I\vec{l} \times \vec{B} = -IlB$

- 도체 막대가 등속도 운동을 위해 필요한 외력 ✓

$\vec{F}_{app} = -\vec{F}_B = IlB$

- 외력에 의해 회로가 공급받는 일률

$P = I\varepsilon = I^2 R = \frac{\varepsilon^2}{R}$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